

Calculators, mobile phones and pagers are not allowed

Answer all of the following questions. Each (sub)question is worth 4 points

1. Let  $f(x) = \tan^{-1}(e^{2x} + 1)$ . Find the domain of  $f$  and show that  $f$  is one-to-one. Also compute  $f^{-1}(x)$  and state the domain and the range of  $f^{-1}$ .

2. Find the limit

$$\lim_{x \rightarrow 0^+} \frac{\ln(\arcsin x)}{\ln(\sin x)}$$

3. Evaluate the following integrals:

(a)  $\int \frac{\ln x}{\sqrt{x}} dx$

(b)  $\int \frac{\sqrt{1+t^2}}{t} dt$

(c)  $\int \sin^3 x \cos\left(\frac{x}{2}\right) dx$

(d)  $\int \frac{dx}{2 - \cos x}$

(e)  $\int_0^{\ln 2} \frac{dx}{\sqrt{e^x - 1}}$  (if it exists).

4. Identify and sketch the curve whose polar equation is

$$r = \frac{5}{3 + 2\sin\theta}$$

by transforming it to rectangular coordinates.

5. Find the area of the region that is inside the circle  $r = 4\cos\theta$  and outside the limaçon  $r = 3 - 2\cos\theta$ .

6. Find parametric equations for the line passing through the point  $P(1, 2, -1)$  which is parallel to the intersection of the planes  $x + y + z = 5$ ,  $x - 2y - 2z = -1$ .

Total 40 points